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# MATHEMATICAL MODEL OF VACUUM-OSCILLATING DRYING OF LUMBER

**R. G. Safin, V. A. Lashkov, L. G. Golubev, and R. R. Safin**

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*A mathematical model of heat and mass transfer in vacuum-oscillating drying of lumber in which the drying process is made up of alternating stages of warming up and vacuum treatment has been developed. At the stage of warming up, it has been proposed to use a superheated steam to intensify heat- and mass-transfer processes and to relax stresses occurring at the stage of vacuum treatment.*

In recent years, scientific research in the wood-working industry has been aimed at producing high-quality wood with a maximum reduction in the duration of the process of drying and decrease in the energy consumption. Thus, vacuum-convective drying chambers [1] ensuring a high quality of drying of lumber satisfy the above requirements to the greatest extent. The drying process in chambers of this type is made up of alternating stages of heating and vacuum treatment. In the process of heating of wood, the material is blown by hot air at atmospheric pressure. Once the material temperature increases to the required value, the process of decrease in the pressure accompanied by the intense evaporation of moisture begins. The drying occurs not only under the action of the pressure gradient but using the temperature gradient as well, which ensures considerable intensification of the process of removal of moisture.

The analysis of the literature sources [2–4] has shown the expediency of the use of a superheated steam at the stage of warming up of the material in the absence of an inert gas in the process of vacuum-oscillating drying. The high rate of heating of the wood as a consequence of the minimum inertia of this stage decreases the total duration of the process of drying. Furthermore, in this regime, a high quality of lumber is attained; such a quality is attributed to the relaxation of stresses that occurred at the stage of vacuum treatment due to the equalization of the moisture content over the thickness as a consequence of the condensation of the steam on the lumber surface at the beginning of the stage of warming up.

In this case, the drying process begins with the creation of vacuum in the drying chamber (see Fig. 1). The process of decrease in the pressure ceases on attainment of a vacuum depth of 95–98 kPa. Next is the stage of warming up, which begins with the injection of steam into the drying chamber and consists of two periods: (1) warming up of the material in the presence of the phase transitions of the heat-transfer agent and (2) convective warming up of the material in the medium of a superheated steam.

The steam arriving at the drying chamber increases the pressure of the medium in the first period of warming up and is condensed on the cold surface of the material. The energy loss by warming-up of the chamber is disregarded as a consequence of the creation of the internal heat insulation or the heated contour of the apparatus. In this case, the material balance can be represented as follows:

$$dm_{\text{ent}} = dm_{\text{st}} + dm_{\text{c}} \, . \tag{1}$$

The amount of steam arriving at a large volume by outflow under high pressure with account for the formula of [5] is found from the expression

$$dm_{\text{ent}} = \mu' \frac{\pi d^2}{4} \sqrt{\frac{2k}{k+1} \rho_{\text{st.g}} P_{\text{st.g}} \left[ \left( \frac{P_{\text{st}}}{P_{\text{st.g}}} \right)^{2/k} - \left( \frac{P_{\text{st}}}{P_{\text{st.g}}} \right)^{(k+1)/k} \right]} \, d\tau \, . \tag{2}$$

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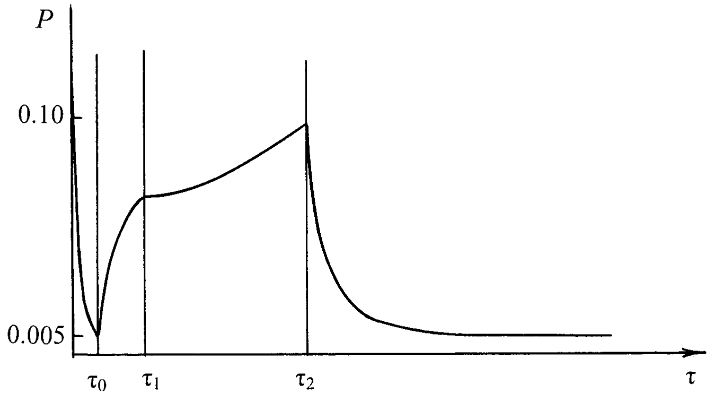


Fig. 1. Cyclogram of the process of vacuum-oscillating drying of lumber. $P$, MPa; $\tau$, sec.

The steam condensed on the material surface $dm_c$ forms a condensate film which fills the capillaries at an infinitely small thickness, and no further diffusion of the steam into the capillaries occurs. The propagation of the moisture into the material is affected by the resistance of the colloidal capillary-porous structure of a body. For anisotropic materials, such as wood, the resistance to transfer in the radical direction is much higher than the resistance to transfer in the axial direction. Furthermore, the low rate of increase of the pressure in the chamber and the condensation of a portion of the steam lead to an insignificant pressure difference over the thickness of the material. On this basis, we take the zone with filled capillaries to be fixed; the change in its thickness can be calculated according to the procedure of [6]. The integral temperature and moisture content of the material are determined with allowance for the existence of different zones.

To describe the heat and mass exchange behind the boundary of the moist zone we use the differential equations obtained by A. V. Luikov [7]; as applied to a one-dimensional symmetric plate and for a single fluid, these equations can be written in the form

$$\frac{\partial U}{\partial \tau} = a_m \left( \frac{\partial^2 U}{\partial x^2} \right) + a_m \delta \left( \frac{\partial^2 T_{mat}}{\partial x^2} \right), \tag{3}$$

$$\frac{\partial T_{mat}}{\partial \tau} = a_t \left( \frac{\partial^2 T_{mat}}{\partial x^2} \right) + \varepsilon \frac{r}{c_{mat}} \frac{\partial U}{\partial \tau}. \tag{4}$$

In the absence of phase changes inside the plate, the criterion of steam generation $\varepsilon$ in (4) is equal to zero. Then the differential equation is reduced to the heat-conduction equation of Fourier

$$\frac{\partial T_{mat}}{\partial \tau} = a_t \left( \frac{\partial^2 T_{mat}}{\partial x^2} \right). \tag{5}$$

The system of equations (3) and (5) is calculated with the following boundary conditions:

$$\frac{r dm_c}{F d\tau} = - \lambda \left. \frac{\partial T_{mat}}{\partial x} \right|_{x=0}, \tag{6}$$

$$U_{surf} = \frac{\rho_{fl}}{\rho_m} \frac{\varepsilon'}{1 - \varepsilon'}, \tag{7}$$

$$U(0; x) = U_0\,, \tag{8}$$

$$T_{\text{mat}}(0; x) = T_{\text{mat0}}\,. \tag{9}$$

The process of feeding of the steam lasts until the absolute pressure (70–75 kPa) inside the chamber is attained and corresponds to the time interval $\tau = \tau_1$, after which the next period of heating begins, which is characterized by the removal of moisture from the material. The warming up of the material under the conditions of forced motion of the superheated steam is described by the system of equations (3) and (5) with the following initial conditions:

$$\left.\frac{\partial T_{\text{mat}}}{\partial \tau}\right|_{\tau=\tau_1} = a_{\text{t}} \frac{\partial^2 T_{\text{mat}}}{\partial x^2}\,, \tag{10}$$

$$\left.\frac{\partial U}{\partial \tau}\right|_{\tau=\tau_1} = a_{\text{m}} \frac{\partial^2 U}{\partial x^2} + a_{\text{m}}\delta \frac{\partial^2 T_{\text{mat}}}{\partial x^2}\,. \tag{11}$$

The boundary conditions for the second period of warming up of the lumber are written by the expressions

$$\alpha\,(T_{\text{st}} - T_{\text{surf}}) - j_{\text{surf}}\, r = -\lambda \left.\frac{\partial T_{\text{mat}}}{\partial x}\right|_{x=0}\,, \tag{12}$$

$$j_{\text{surf}} - \beta\,(\rho_{\text{surf}} - \rho) = 0\,, \tag{13}$$

the moisture flux to the mass-exchange surface is determined by the relation

$$j_{\text{surf}} = \rho_0 \left( a_{\text{m}} \left.\frac{\partial U}{\partial x}\right|_{x=0} + a_{\text{m}}\delta \left.\frac{\partial T_{\text{mat}}}{\partial x}\right|_{x=0} \right). \tag{14}$$

After the stage of warming up ending in the attainment of a temperature of 338–343 K at the center of the material, the process of decrease in the pressure begins, for which the equation of material balance for the steam is written as follows [8]:

$$j \cdot 2\,(h + b)\, l d\tau - V_{\text{s.st}}\, \rho_{\text{st}} d\tau = V_{\text{fr}} d\rho_{\text{st}}\,, \tag{15}$$

where the density of the steam can be related to the pressure by the Mendeleev–Clapeyron equation

$$\rho_{\text{st}} = \frac{P\mu}{RT} \tag{16}$$

or

$$d\rho_{\text{st}} = \mu \frac{TdP - PdT}{RT^2}\,. \tag{17}$$

Having substituted expressions (16) and (17) into Eq. (15), after certain transformations we obtain the differential equation of a change in the pressure above the lumber:

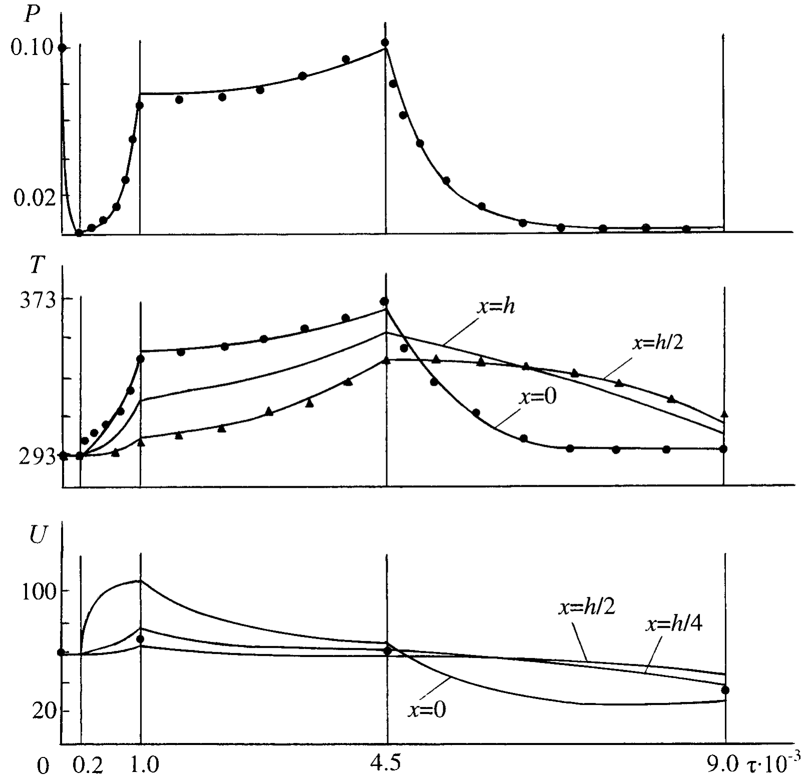


Fig. 2. Experimental data and calculated curves of change in the pressure of the medium and in the temperature and the moisture content of the material in layers. *P*, MPa; *T*, K; *U*, kg/kg; τ, sec

$$\frac{dP}{d\tau}=\frac{2\,(h+b)\,lRT}{V_{\rm fr}\,\mu}\,j-P\left(\frac{V_{\rm s.st}}{V_{\rm fr}}-\frac{1}{T}\frac{dT}{d\tau}\right). \tag{18}$$

The heat and mass transfer in the process of decrease in the pressure is also described by the differential equations (3) and (5) for which the initial conditions can be represented in the form of Eqs. (10) and (11) for the instant of time $\tau = \tau_2$ (which corresponds to the beginning of the stage of vacuum treatment), while the boundary conditions can be represented in the form of the expression [9]

$$U_{\rm surf}=a\left(\frac{P}{P_{\rm sat}}\right)^{n}, \tag{19}$$

$$-\,j_{\rm surf}\,r-\lambda\,\frac{\partial T_{\rm mat}}{\partial x}\bigg|_{x=0}-T_{\rm matsurf}\,c j_{\rm surf}-\alpha\,(T_{\rm matsurf}-T)=0\,. \tag{20}$$

In Eq. (20), the first term characterizes the expenditure of heat on evaporation, the second term describes the heat flux from the depth by heat condition, the third term characterizes the removal of heat with a moisture vapor, and the fourth term takes into account the heat exchange with the vapor phase.

The stage of vacuum treatment lasts until the temperature at the center of the material attains 303–308 K, after which the cycle is repeated. The number of "warmup–vacuum" stages is determined by the thickness of the lumber, the final moisture content, and the kind of wood.

The adequacy of the mathematical model has been tested on an experimental setup which included a heated vacuum chamber connected with a vacuum pump via a shell-and-tube condenser, a steam generator, and devices for

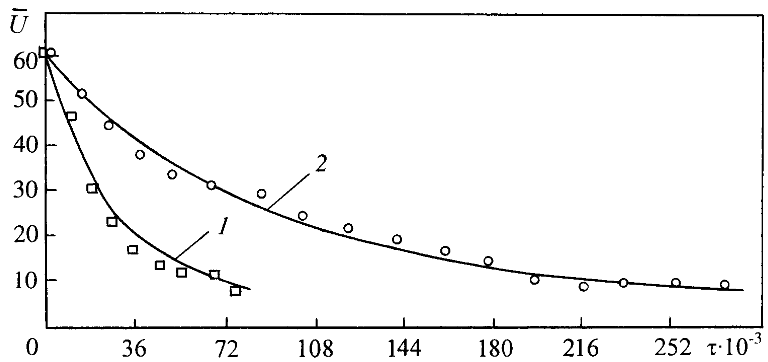


Fig. 3. Curves of drying of: 1) pinewood samples; 2) oak samples. $\bar{U}$, kg/kg; $\tau$/sec.

recording the temperature of the material and the medium and the absolute pressure in the chamber. The above system of differential equations was calculated using the MathCAD Professional software package making it possible to solve the equations by the Runge–Kutta method.

The experimental and theoretical investigations on drying were carried out on samples of different kinds of wood.

Figure 2 presents calculated and experimental data on a change in the temperature and moisture content over the cross section of the lumber and the pressure of the medium in the first cycle of oscillation. Pinewood boards (1500 × 200 × 25 mm) with an initial moisture content of $U = 60 \pm 2\%$ were used as the samples under study. The heating of the wood was carried out at a temperature of the medium of $T_{st}$ = 373–383 K and lasted until the temperature $T_{c.mat}$ = 338–343 K at the center of the material was attained. Thereafter the material was kept to $T_{cent.mat}$ = 303–308 K at a residual pressure of 2 kPa. The calculated values after the stage of warming up, whose duration was 75 min, showed that the temperature at the center of the material increased from 293 to 341 K; the change in the value of the integral moisture content amounted to 13–15%. The process of decrease in the pressure caused a change of 8–18% in the moisture content of the layers.

The results of investigations of the influence of the material structure on the process of removal of moisture are presented in Fig. 3 in the form of drying curves. The analysis of the curves shows that in the denser wood of oak, the rate of drying is more than threefold lower than in the pinewood samples.

The comparison of the experimental data to the calculated ones has shown that the maximum disagreement is no higher than 18%.

## NOTATION

*m*, mass, kg; *U*, moisture content of the material, kg/kg; $\bar{U}$, integral moisture content, kg/kg; $\tau$, running time, sec; $a_m$, mass-diffusivity coefficient; $m^2$/sec; $\mu'$, flow-rate coefficient; $\delta$, relative coefficient of thermodiffusion, 1/K; *d*, diameter of the tube for feeding the steam, m; *k*, adiabatic exponent; *T*, running temperature, K; *P*, pressure, Pa; $\rho$, density, kg/$m^3$; *x*, coordinate; $a_t$, thermal-diffusivity coefficient, $m^2$/sec; $\varepsilon$, steam-generation criterion; $\varepsilon'$, porosity of the material; *r*, latent heat of vaporization, J/kg; *c*, specific heat, J/(kg·K); *F*, cross-sectional area of the material, $m^2$; *h*, *b*, and *l*, thickness, width, and length of the material, m; $\lambda$, thermal-conductivity coefficient, W/(m·K); $\alpha$, heat-transfer coefficient, W/($m^2$·K); $\beta$, mass-transfer coefficient, m/sec; *j*, mass flux, kg/($m^2$·sec); *R*, universal gas constant, J/(kmole·K); $\mu$, molar mass, kg/kmole; $V_{fr}$, free volume of the chamber, $m^3$; $V_{s.st}$, volume capacity of the system of removal of the steam, $m^3$/sec; *a* and *n*, coefficients of Eq. (19). Subscripts: ent, steam entering the drying chamber; st, steam; c, condensate; st.g, steam generator; mat, material; 0, initial instant of time; fl, fluid; surf, surface; cent, center; sat, saturation; eq, equilibrium; m, mass diffusivity; t, thermal diffusivity; s.st, system of removal of the steam; fr, free.

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